A QUADRATIC ASSIGNMENT / LINEAR PROGRAMMING APPROACH TO SHIP SCHEDULING FOR THE U. S. COAST GUARD

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THESIS

A QUADRATIC ASSIGNMENT / LINEAR PROGRAMMING APPROACH
TO SHIP SCHEDULING FOR THE U.S. COAST GUARD

by

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June, 1977

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I. INTRODUCTION

A. BACKGROUND

The United States Coast Guard is responsible for many areas of national concern in the maritime regions of this country. These responsibilities, called "missions" by the Coast Guard, include search and rescue, law enforcement, fishery and custom regulation, small craft and commercial vessel safety, icebreaking, International Ice Patrol and collection of oceanographic data. To coordinate these responsibilities, two major commands have been established - one for the Atlantic, Gulf of Mexico area, and one for the Pacific area. Each area is further subdivided into several districts. The District Commanders handle all operational and administrative matters arising in their districts. The Area Commander, however, assumes operational control for all missions that are "multi-district" in geographical region, scope, or resources. Some of these missions are: law enforcement and fishery patrols in the Alaska region, oceanographic data collection, and readiness training.

To fulfill his responsibilities, the Area Commander must supervise the allocation of resources for these missions. Periodically, the Pacific Area Commander's staff prepares and formally issues the operational schedules for the High and Medium Endurance Cutters in the Pacific Area. This published schedule also satisfies the maintenance, military readiness, and training requirements. It serves as the operational orders to the Commanding Officers of the ships,

specifying where, for what, and when the ships are needed. It informs the crews when they can expect to be home with their families and when and for how long they will be away. The Coast Guard's greatest asset is its people; the schedule has a large impact upon their lives. Thus, the published schedule is of prime concern to all - the Area Commander, District Commander, their staffs, ship's Commanding Officer, and the manning personnel and families.

B. CURRENT SCHEDULING METHOD

In any organization, the scheduling of resources is part of the managerial planning and control function. The areas of responsibility of the Coast Guard, by their nature, generate requirements that can never be completely satisfied. Law enforcement and protection of lives and property are examples of missions that can never be over-performed. The Coast Guard possesses a finite number of resources - ships and personnel. These resources have physical limitations and needs that limit their utilization to fulfill the mission requirements. The managerial planning and control function of the Area Commander is to allocate his resources, striking a balance between the requirements and these constraints.

To furnish the schedulers with basic information concerning what missions are required, what resources are available, and what limitations exist, the Area Commander has established a set of guidelines. These guidelines compile Coast Guard-wide, Area, and District policy decisions and plans by specifying patrol standards, lengths and frequencies for each mission. Resource limitations are expressed as annual maintenance and training requirements. The needs of the manning personnel are reflected in workload

balancing, maximum cruise lengths, minimal times to be spent in homeport, and other items that affect the crew's "morale".

These morale-related limitations result in goals that the scheduler attempts to satisfy. The evaluation and comparison of two different schedules that satisfy the mission requirements and morale-related goals in different balances can not be performed using strictly objective criteria. Judgement and individual preferences are involved.

Using the guidelines, two schedulers spend about one week manipulating a large magnetic Gantt chart and calculating summary totals for each candidate schedule (e.g. total time away from homeport, total travel time for each ship). Then, additional time is spent with the resultant schedule in negotiations between the decision makers, the schedulers, and the Commanding Officers of the ships. The desired final schedule is one that satisfies the mission requirements with consideration and respect for the morale-related goals.

Each January and July a formal schedule is published covering the next 18 month period. A tentative schedule for the subsequent six month period (months 19 to 24) is also prepared, but distributed informally. The final published schedule provides one-day time resolution, specifying required on-scene arrival and departure dates and estimated homeport departure and return dates.

C. GOAL OF THIS STUDY

The immediate goal of this exploratory research is the development of an analytic scheduling model based on

mathematical programming techniques. This analytic model, incorporating the most important of the mission requirements and morale-related goals, will generate potential working schedules using a reasonable amount of computer resources. These potential schedules will present the decision makers and schedulers with more choices for objective and subjective evaluation than the current manual method. This managerial tool will aid strategic and tactical decision processes, and possibly improve them by relieving a heavy clerical workload.

Historically, scheduling problems (as an application of mathmatical programming) have either been oversimplified with a resultant loss of effectiveness, or so detailed that computational difficulties prevent economical solution. Certain recent advances in optimization capabilities, however, have made possible this exploratory research to apply mathematical programming models to the Coast Guard scheduling problem.

The chapters of this thesis follow a systems analysis approach:

1. Formulation (identify problem and objectives),
2. Analysis (examine elements, interrelationships, variables, and constraints),
3. Synthesis (classify the problem and study alternative methods of solution),
4. Selection of Method and Detailed Modeling,
5. Implementation (the test of all that precedes), and
6. Evaluation and Reformulation.

II. ANALYSIS OF SCHEDULE AND SCHEDULING GUIDELINES

A. SCHEDULE'S MEASURE OF EFFECTIVENESS

Unlike typical production or job shop scheduling, the Coast Guard problem does not possess a single measure of effectiveness on which objective evaluation of candidate schedules can easily be based. There are some objective evaluation criteria. However, the subjective criteria derived from the morale-related considerations play the dominant role in evaluation. Calendar-related events are also considerations (e.g. fishing seasons, major holiday periods, and weekends). Individually, each requirement or morale-related goal is reasonable. Unfortunately, there is no schedule that can simultaneously meet all the desired goals. Since any schedule will have violations of the guidelines, it is necessary to assess the trade-offs among the conflicting requirements, and to balance the various goals. Managerial judgement and personal preferences (of the Area and District Commanders, their staffs, and each ship's Commanding Officer) ultimately determine the properties and structure of the final schedule.

B. BASIC ELEMENTS

There are three basic elements in the schedule - ships, time, and missions. There are presently seven High Endurance Cutters (HEC) and five Medium Endurance Cutters

(MEC) in the Pacific Area that are scheduled by the Area Commander. An eighth HEC arrives 1 June 1977. The "Hamilton class" of HEC's, because of their different capabilities, are used for different types of missions than the single older HEC in the Area. Scheduling interactions between these ship types and all other ships are minimal. Thus, the scheduling process can be divided into two separate cases - one for the "Hamilton" class HEC's and one for the other, smaller cutters. To limit scope, and emphasize basic issues, this study concentrates on the scheduling process for just the "Hamilton" High Endurance Cutters (HEC's). Two of these HEC's are based (homeported) in Honolulu, Hawaii; two in San Francisco, California; and the rest in Seattle, Washington.

As mentioned above, each published schedule covers an eighteen month period. Months 19 to 24 are tentatively scheduled but not formally distributed. Between successive schedules, there is a desire to minimize the number of major changes. Unless major changes in mission requirements have occurred or unforeseen events have taken place, the first six months of a new schedule will be in close agreement with the same period for the previous schedule. As the time frame of the schedule progresses, more changes and thus greater deviations occur. The informal 19 to 24 month segment is the initial projection for this calendar period; it is, therefore, highly speculative.

The missions performed by the "Hamilton" class High Endurance cutters cover most of the Coast Guard's areas of responsibility - fishery and custom regulation, law enforcement, oceanography, search and rescue, military readiness, and personnel training. Two basic categories of requirements originate from these responsibilities. The first type will be termed SHIP-SPECIFIC requirements. These assignments arise from requisites physically associated with

a particular ship and her crew. There is a need to maintain her equipment and machinery, and train her crew to function as a unit so that the ship can perform general operational missions. For example, Refresher training (Reftra), Navy ASW exercises (Navex), and Maintenance periods (MAINT) are SHIP-SPECIFIC requirements. Each ship must be scheduled to individually satisfy these needs. The second type of requirements, termed GENERAL, cover the direct operational areas of responsibility. Successions of ships are used to collectively satisfy these requirements. Alaska fishery patrols (Alpat), Academy Cadet training, and oceanographic data collection (Ocean) are examples of GENERAL requirements.

The geographical area where the mission is performed is also an important consideration. Travel time to and from the mission area is necessarily present in the one-day resolution of the published schedule. (For instance, travel time from Hawaii to Alaska is 7 days.) The time spent by a ship away from its homeport is affected by this travel time. Also, in order to minimize travel time to the mission area, different missions are performed by the Hawaii-based ships than by the continental-based (CONUS) ships.

C. GUIDELINES

The GENERAL and SHIP-SPECIFIC requirements are specified in three ways by the scheduling guidelines. The first is the setting of the <u>number</u> of ships that are to be on-scene in a given area at a given time (e.g. During May, two ships should be near the Aleutian Islands). This type of specification is called a PATROL requirement. The second is the setting of a repetitive <u>frequency</u> to be met (e.g. once per quarter). The fulfillment of this type of requirement

usually takes a standard amount of time and is to be accomplished without interruption. The third is the specification of a <u>total amount of time</u> to be spent in a given period. This time quota will usually be divided into several segments (e.g. The maintenance requirement of 13 weeks per year may be divided into segments of 4, 4, and 5 weeks each.)

Three principal morale-related guidelines affect the amount of time each ship can be scheduled to be away from its homeport. First, a maximum limit is set for the total "Away Homeport" (AHP) time per year for each ship. Second, it is desirable to balance AHP time between ships since it functions as a pseudo-measure of each ship's share of the workload. Third, a maximum limit is set for the duration of any single cruise (the time from departure from homeport to return).

Other guidelines concern the longest and most difficult GENERAL requirement - Alaska Fishery Patrol (Alpat), A maximum limit on the total yearly Alaska Patrol time per ship is set. If a ship is assigned back-to-back Alpat missions, the minimum intervenung in-homeport pericd is eight weeks. Before any Alpat, a minimum four week inport period is desired for adequate preparation. It is also desired to alternate ships that are assigned Alpat missions in the rough weather months of winter.

III. CHARACTERIZATION OF PROBLEM AND MODEL SELECTION

A. CHARACTERIZATION

Examination of the published schedules for the period January 1975 to December 1976 shows the following basic elements: 6 ships, 2 year time span, 6 areas of responsibility generating requirements, and 129 separate missions used to satisfy the requirements.

The frequency of mission-to-mission transitions made by the ships is shown in Figure 1.

Mission to Mission Transitions
January 1975 to December 1976

From:	To: Inport	Alpat	Ocean	Reftra	Navex	Maint
Inport		3	12	7	5	27
Alpat	11		4			1
Ocean	4	14				
Reftra	7					1
Navex	6			1		1
Maint	15	2	2		4	1

Figure 1

For instance, ships made the transition from Oceanographic data collection (Ocean) to Alaska Patrols

(Alpat) fourteen times in the two years.

The schema used by Conway, Maxwell, and Miller [3] to describe scheduling problems is a useful classification and helps to succinctly define which factors are known and unknown by the scheduler:

1. Requirement Arrival Process - At the start of the scheduling process, the total requirements for each area of responsibility are specified within a given expected range. The number of separate missions that will be necessary to fulfill each requirement is not specified prior to the scheduling process. Requirements are not simultaneously available but become available at individual times according to frequencies and timings given in the guidelines.

2. Resources Available - The number and individual capabilities of all ships are known by the scheduler.

3. Flow Pattern - As shown in Figure 1, the mission transitions that occur have definite patterns. The sequencing of missions is critical, mostly because of the limitation on cruise length. Also, the PATROL requirement introduces precedence and linking relationships between the individual missions used to fulfill the requirement: the end of a patrol for one ship should implicitly specify the start of a patrol for another ship.

4. Measure Of Effectiveness (MOE) - As explained previously, there is no single MOE but rather a complex combination of requirements and goals.

This problem also possesses constraints beyond the scope of those given by Conway, Maxwell, and Miller. These constraints are caused by the physical limitations of the

ships and the needs of their crews. The total AHP time scheduled for each ship should be equally balanced with respect to the other ships. The length of single cruises should be less than the maximum limit of the endurance of the ship or crew. For each ship, the total Alaska Patrol time per year should not exceed the specified limit. These three constraints, referred to as the "limiting" constraints in further discussions, are not typical production or job shop constraints.

To summarize the Coast Guard problem, the scheduler has definite resource information and definite knowledge of the type of transitions that can occur. He knows the total mission area requirements and must determine the number and durations of the separate missions that collectively satisfy these requirements. The requirements are not simultaneously available at the beginning of the period but arrive dynamically. The "limiting" constraints impose additional restictions on the sequence and durations of separate missions.

This problem is very large. The final schedule has one-day resolution for about 5,000 ship-days and about 6 mission-types. Any economically feasible analytic model must aggregate and divide the schedule period into weeks or even months and then schedule in terms of these units.

B. SELECTION OF APPROACH

An Integer Linear Programming approach has been considered. This approach to scheduling problems (as demonstrated by Prabhaker [10]) is not feasible because of the large scale of this Coast Guard problem. A schedule for 6 ships, 6 mission-types and 2 year time span with 1 week

resolution would yield about 3,600 0/1 variables. This size is too large for commercially available computer codes. A special code for this particular application would have to be developed. (There is no apriori guarantee that even the best possible approach would solve the problem with a reasonable amount of computer resources.)

Simulation has been discarded as a solution method for several reasons. Long development time is required to implement the proper sequence of random starting solutions and priority or heuristic-guided search methods. (Panwalkar and Iskander [9] list over 100 static and dynamic scheduling or dispatching rules.) This approach is typically expensive to run. Difficulties exist in interpreting the results: data sensitivity and experimental variation are difficult to differentiate. Effective external controls for guidance or model coersion are usually lacking. There are also serious reservations about the potential flexibility of the resultant computer program to meet future problem changes (in the guidelines, for example).

A foundation of optimization theory and techniques is desired. The general approach initially selected to address this Coast Guard scheduling problem is the hybrid Quadratic Assignment/Linear Programming model of Geoffrion and Graves [4]. The detailed description of this model is presented in the next Chapter but a brief summary is appropriate here.

A Quadratic Assignment model determines a candidate sequence of separate missions for each ship. A Linear Programming model then determines the duration of each mission. Then, two heuristic searches vary the candidate sequence and new durations are obtained in an attempt to improve the schedule.

This model has been selected for the reasons listed below:

1. The flexibility in the problem statement accomodates nicely the indefinite information on the number and duration of individual missions.

2. No simplifying assumptions or deletion of guideline restrictions appear to be necessary, allowing an uncompromising view of the entire scheduling problem.

3. The Quadratic Assignment model naturally handles the sequencing restrictions (which normally cause great difficulty and computational complexity for analytic models).

4. The continuous Linear Programming model naturally handles the "limiting" constraints of this problem and allows the lengths of the missions to be optimally adjusted to best meet the guidelines.

5. The Linear Programming model (using flexible penalty functions) is a natural way to incorporate the guidelines and morale-related goals into a linear objective function.

6. The model handles dynamic, non-simultaneous arrival of requirements.

7. The model pcssesses special structure amenable to implementation techniques appropriate for the large scale of this problem.

8. This hybrid model has been successfully implemented for an industrial production application of smaller scale.

IV. GEOFFRION-GRAVES MODEL

A. OVERALL PHILOSOPHY

The model proposed by Geoffrion and Graves makes several significant contributions to solving an interesting a class of scheduling problems: the separation of the combinatorially difficult sequencing problem from the continuous allccation and timing problem; a flexible problem statement reflecting true managerial discretion; and a method to express this class of scheduling problems in the rigid, finite formulation required for the Quadratic Assignment problem.

The class of scheduling problems addressed by this hybrid method is stated in production terminology as follows:

> There are several "similar" continuous process facilities (lines) operating in parallel. Each is able to manufacture some subset of products with known production rates and costs. Significant costs are incurred by each changeover of a line from producing one product to another. Production orders are received dynamically and are not simultaneously available for scheduling. Each order is for a specific total amount of production to occur between an early start time and a late finish time. An order can be split among lines or produced non-contiguously on the same line. The

desired schedule has the minimum total production and transition costs over a specified horizon.

The solution approach followed by their model allows the broadening of this problem statement to allow lower and upper limits on the order demands rather than an exact amount. Also, violation of the start and finish times of orders, and early or late completion by individual lines is allowed but penalty costs are incurred.

In the hybrid model, a Quadratic Assignment (QA) problem is formulated and solved using the exact demands, the specified horizon, and the early start and late finish times. Violations of the horizon or start/finish times incur penalty costs. The QA solution is not an optimal, minimal cost solution. The problem is solved by heuristic methods that obtain very good, low cost final results that are "locally optimal" (i.e. certain easy changes made to the solution cause increased cost). For the remainder of this thesis, "minimal" implies this locally optimal condition. The resultant assignment (lines to orders over the fixed time span) is then used only for the sequence of orders assigned to each production line. Each continuous production run on a line, called a "campaign," is noted. The quantity and timing information is discarded.

Given the product sequence from the QA solution, a Linear Program (LP) is generated and solved to find the duration of each campaign on each line, using the bounded demands, the desired early start and late finish times, and a flexible schedule horizon. The LP minimizes total production and penalty costs (incurred by violations of the desired order times and/or horizon). The sequence costs are determined by the QA sequence and are not changeable by the LP. Next, two local searches are employed to examine the sequence of campaigns and locate potentially better product

sequences. For each favorable candidate sequence, another LP solution is obtained. The final solution is that combination of sequence and product durations with the minimum total production, sequence and penalty costs.

B. QUADRATIC ASSIGNMENT FORMULATION

Although many variants of the Quadratic Assignment problem have been presented in the literature, this discussion covers the models influencing the Coast Guard QA model. Discussed are the original Koopmans-Beckmann model [7], the generalization addressed by Graves-Whinston [6], the specialization of Graves-Whinston used by Geoffrion-Graves in their scheduling model, and (in the next Chapter) the large scale QA model arising in the Coast Guard problem. The reader will note that the specializations made in the Coast Guard model are appropriate to this particular problem. The techniques to handle the large scale can be applied to generalizations of the Coast Guard model with some additional effort.

1. General Koopmans-Beckmann Formulation

The original statement of a Quadratic Assignment (QA) problem was made by Koopmans and Beckmann for the assignment of n "indivisible" manufacturing plants to n fixed geographical locations so that inter-plant transportation costs of one commodity are minimized. As stated by Lawler [8], let

c_{jk} = transportation cost per unit from location j to location k;

q_{ip} = quantity shipped from plant i to plant p (interaction quantity);

x_{ij} = 1 if plant i is assigned to location j,
= 0 otherwise.

The object is to minimize the interplant transportation costs,

$$\sum_{p=1}^{n} \sum_{i=1}^{n} q_{ip} \left[\sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk} \, x_{ij} \, x_{pk} \right], \tag{1}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for} \quad j = 1,n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for} \quad i = 1,n \; .$$

That is, the optimal pairs of (plant i, location j) are desired.

2. Graves-Whinston Generalization

Graves and Whinston have generalized the Koopmans-Beckmann model by the addition of a fixed cost term to the objective function. The assignment of plant i to location j can incur known, fixed costs (for example, purchase of land, installation of highways and other services). The problem is to minimize total transition plus fixed costs in determining the (plant, location) pairings.

3. Geoffrion-Graves Formulation.

The Geoffrion-Graves formulation is a specialization of the Graves-Whinston QA problem. Restrictions are placed on the "plant"-to-"plant" interactions that occur.

In the G-G QA formulation, the scheduling horizon (0,H) on each production line is divided into equal indivisible time-units. Each time-unit is a "plant" in the Koopmans-Beckmann sense. The interaction between time-units (plants) affects only those time-units immediately preceeding and following them on the same production line. The interaction Q matrix (composed of all q_{ip}) has the structure shown in Figure 2.

Q-Matrix

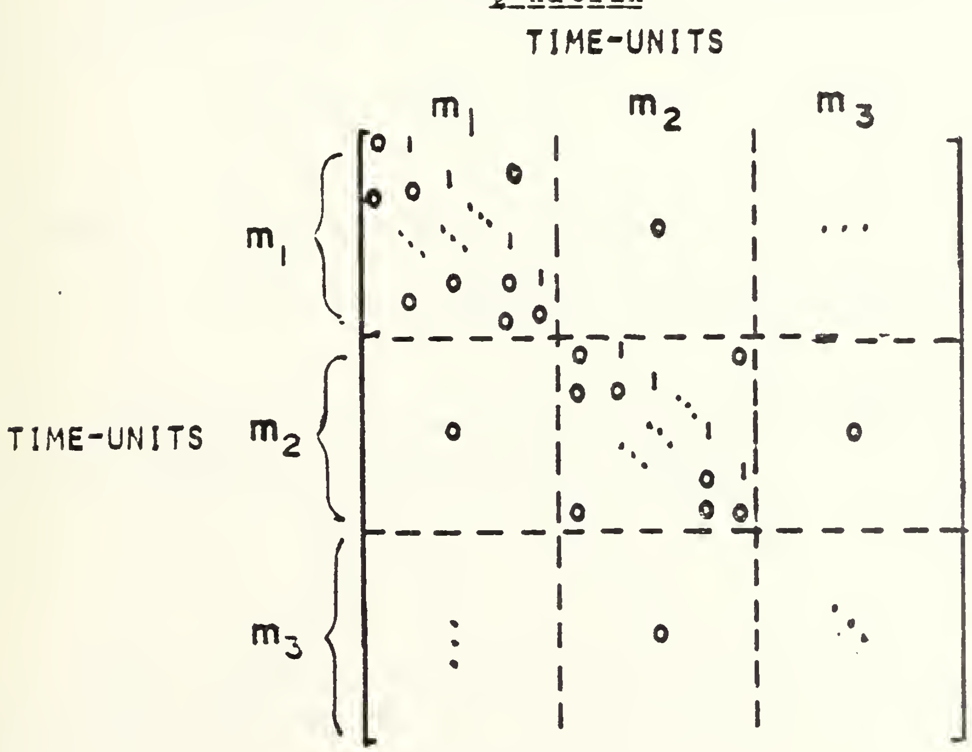


Figure 2

(Note: The Geoffrion-Graves model is generalized to handle

"similar" production lines. Identical production and transition costs between lines are not required. Proportionality between lines for these costs may exist. The +1 entries in the Q-matrix are replaced by the line's transition cost proportionality constant.)

The quantity A of each production order is divided into equal indivisible "product-units." Each product-unit is a Koopmans-Beckmann "location.".

The finite quantization is determined by the choice of the basic time resolution S. The number of time-units per line l is:

$$m_l = H / S, \tag{2}$$

where H = the schedule horizon.

The number of product-units per production order k for a product p is:

$$n_k = A_k / (R_p * S), \tag{3}$$

where A_k = the order's specified demand

R_p = the product's standard production rate.

Requiring that the total number of time-units equal the total number of product-units yields:

$$n = \sum_l m_l = \sum_k n_k . \tag{4}$$

The problem is infeasible if total product-units exceed total time-units. A "slack" product can be added to enforce (4).

The quadratic assignment problem can now be stated as the set of pairings (time-unit i, product-unit j) such that total transition costs plus total fixed costs is minimized,

$$\sum_{p=1}^{n} \sum_{i=1}^{n} q_{ip} \left[\sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk} \, x_{ij} \, x_{pk} \right] + \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \, x_{ij}$$

subject to (5)

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for} \quad j = 1,n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for} \quad i = 1,n$$

where:

c_{jk} = transition cost between product-units j and k;

q_{ip} = 1 if time-units i and p interact,

= 0 otherwise;

d_{ij} = fixed cost of assigning product-unit j to time-unit i;

x_{ij} = 1 if product-unit j is assigned to time-unit i,

= 0 otherwise.

The C matrix, shown in Figure 3, contains the product-unit to product-unit transition cost, derived from the standard product-to-product costs. Each product-unit is associated with an order which is for a particular product. (The standard transition cost between product p and product

p' is denoted by $T_{pp'}$.)

<u>C - Matrix</u>

MISSION-UNITS

$$
\begin{array}{cc}
 & \begin{array}{cccc} n_1 & n_2 & \cdots & \cdots \end{array} \\
\text{MISSION-UNITS} \quad \begin{array}{c} n_1 \\ n_2 \\ \vdots \\ \vdots \end{array} & \left|\begin{array}{cccc}
0 & T_{p_1p_2} & T_{p_1p_3} & \cdots \\
T_{p_2p_1} & 0 & T_{p_2p_3} & \cdots \\
T_{p_3p_1} & T_{p_3p_2} & 0 & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{array}\right|
\end{array}
$$

Figure 3

The d_{ij} form the D matrix of fixed costs. This matrix contains costs (incurred when product-unit j is assigned to time-unit i) which account for the following effects:

1. The production costs;
2. The transition cost, for each line, from the last product on the previous schedule to the first in this scheduling period;
3. Prohibitive penalties for producing an order before the

early start time or after the late finish time;

4. Infeasibility of certain products which cannot be produced by certain lines (a high cost of infeasibility is used).

(Note: There is an explanation of how this QA formulation is interpreted for ships and missions of the Coast Guard problem in the next Chapter. Also, Appendix A contains an example and picture.)

C. LINEAR PROGRAM FORMULATION IN G-G MODEL

A change of variables occurs between the (time-unit, product-unit) pairs of the QA and the time durations of the LP. Each continuous production run of a product (product-units for the same order) is noted and called a "campaign." The primary LP variables are the durations of each campaign.

Without changing this sequence of campaigns on each line, the LP determines the duration (in continuous time) of each campaign so that the minimal total cost is obtained. The three cost components in the LP's objective function are:

1. Production costs,
2. Penalties for violating orders' early start or late finish time, and
3. Penalties for modifying the specified horizon, H, on a line.

The total quantities for an order are constrained to fall between the specified lower and upper limits.

D. IMPLEMENTATION.

Geoffrion and Graves have implemented their model to schedule six chemical reactors for a one month planning horizon for Dart Industries, Inc. Various kinds of plastics are the products. The QA problem is solved by the Graves and Whinston method [6].

The Graves and Whinston method solves a generalization of the Koopmans-Beckmann QA problem. To simplify this discussion (and without loss of generality), the special structure of the Q-matrix of the G-G model is assumed. The G-W method is an n-stage decision process where n is the total number of time-units for assignment. At the k^{th} stage, k-1 pairings of (time-unit i, product-unit j) have been made. In the basic method, no backtracking occurs so these k-1 assignments are permanent. The k^{th} stage examines all possible pairings of the n-(k-1) unassigned units. For each pairing, the expected final total cost, comprised of three components, is calculated. These three components are:

1. The costs incurred by the previous k-1 pairings,
2. The immediate cost of assigning the pair being considered at this k^{th} stage, and
3. The expected value of future costs that will be incurred by the remaining n-k unspecified assignments, given that the current pair is made permanent.

The expected value of future costs is calculated explicitly by a formula rather than by enumeration. The

assumption is made that the assignment probability of each possible permutation of assignments is equal. The replacement of enumeration for unspecified assignment costs by the conditional expected value is a significant contribution of the G-W method.

The pair, designated by (i', j'), with the minimum expected total cost is selected by the assignment criteria. Next, the fixed costs, d_{ij}, of those time-units that immediately preceed or follow i' are updated. For example, let time-unit i preceed i'. Any product-unit j assigned to time-unit i will be followed by product-unit j'. The transition cost of j to j' is now known and is thus a fixed cost. The fixed cost elements for time-unit i are thus updated by the addition of this known transition cost. A similar update for the time-unit following i' is also done. The method then proceeds to the k^{th} stage.

Since no conditional computations occur at each stage, the computational effort and memory requirements of this portion of the G-W method may be determined as a simple function of n.

A local search (called Switch) is next performed on the (time-unit, product-unit) pairings. All pairwise exchanges of product-units are cyclically tested for possible improvement in total cost. For each new pairing, the change in fixed costs and transition costs is deterministic.

At the termination of this Switch algorithm, a QA solution is obtained. Next, the sequence of campaigns is extracted and made the incumbant sequence for the LP stage of the G-G model. A Linear Programming solution is obtained for this sequence. Then, two different local searches are

performed on the candidate sequence. The first search (called Slide) moves each campaign into all other possible positions. The second search (called Switch) makes a pairwise interchange of all campaigns. For each new, favorable sequence of campaigns selected by a screening criteria, an LP solution is obtained. If the total cost (sequence plus LP costs) is improved, the candidate sequence replaces the incumbent and the two local searches are restarted. When both searches terminate, the incumbent sequence and durations constitute the "locally" optimal, final solution.

The LP problems are solved by G-G with an LP subroutine described in Graves [5]. This algorithm includes advanced features for dealing with degeneracy, employs a complete mutual primal-dual simplex mechanism, and uses a basis representation with a small, segregated inverse. The program uses complete in-core storage of the coefficient matrix, and handles logical variables implicitly. Ranging of equations and bounded variables are not available.

V. ANALYTIC MODEL FOR COAST GUARD PROBLEM

A. OVERALL CONSIDERATIONS

The Coast Guard scheduling model is based on the Geoffrion and Graves approach. The Area Commander's scheduling guidelines, expressing mission requirements and the morale-related goals and constraints, are converted into the objective functions and constraints of the Quadratic Assignment and Linear Programming models. The objective function expresses pseudo-costs that are incurred by deviations from the requirements and goals. The optimization models, by minimizing their objective functions, seek the closest compliance of the resulting schedule to the guidelines. The objective functions are summations, each term expressing with penalty costs the degree of compliance with one of the requirements or goals.

In the Coast Guard problem, a ship corresponds to a G-G "line." The time available on a ship is divided into discrete time-units. Each mission requirement is a G-G "production order" and is similarily divided into mission-units.

The guideline's requirements and goals have not been completely captured by the QA and LP models. Some features are included in the QA that are not in the LP and vice versa. The guidelines not modeled at all are left for human action upon the resultant schedule.

B. QUADRATIC ASSIGNMENT MODEL

The modeling for the Quadratic Assignment algorithm is performed by setting the structures and components of the three matrices:

- C - mission-unit to mission-unit transition costs,
- Q - interaction between time-units, and
- D - fixed costs.

Each of these matrices are specially structured for this particular Coast Guard problem. The C-matrix will coerce the mission sequences to follow the actual patterns shown in Figure 1. The Q-matrix is the same as the G-G matrix. The D-matrix will express in fixed costs the desired start/finish times, the two types of mission requirements - PATROL and single continuous duration, and other items.

1. C - Matrix

The mission-unit to mission-unit transition costs are directly determined from the requirement-to-requirement transition costs. Frequently occurring mission transitions are strongly encouraged by low costs and undesired transitions are discouraged by high costs. (A transition between mission-units of the same requirement incurs no cost.) Figure 1 in Chapter III shows the frequency of the various transitions in actual schedules. The mission Inport (ship in homeport on search and rescue standby) plays the dominant role. Most of the time, Inport is the mission occurring between all other missions. The exception, Ocean (oceanographic data collection), is performed enroute or returning from Alaska Patrol. The number of occurrences of

each extended sequence is shown below.

Inport/Ocean/Alpat	14 (approx)
Alpat/Ocean/Inport	4

With only a "from Mission A to Mission B" transition cost, a difficulty arises in modeling these two alternatives. The goal of the model is to match the scheduler's logical ranking with the ranking determined by the pseudo-costs structure. The example below shows the problem.

From-To Transition Difficulty

Sample Transition Costs:

		To:		
		Inport	Alpat	Ocean
From:	Inport	0	30	10
	Alpat	10	0	30
	Ocean	90	10	0

Sample Sequence:	Logical Rank	Pseudo-Cost Rank	Transition Cost
Inport/Ocean/Alpat/Inport	1	1	30
Inport/Alpat/Ocean/Inport	2	3	150
Inport/Ocean/Inport	3	2	100

Figure 4

The sequence Inport/Ocean/Inport would never be scheduled and thus is highly unfavorable. The pseudo-cost mechanism can not convey this as the difference in the rankings shows. The "from A to B" structure cannot handle "from A to B to C" sequences. A possible way to prevent the undesired and unrealistic Inport/Ocean/Inport sequence from

having an inappropriately low cost is to eliminate Ocean/Alpat or Alpat/Ocean. Based on the frequency Table, Ocean/Alpat is retained.

The same difficulty occurs if Travel time is modeled as a mission to preceed or follow those missions requiring it. The irrational sequence Inport/Travel/Inport would have a lower cost than a logically prefered sequence. (e.g. Inport/Travel/Reftra/Travel/Inport). Thus, Travel time as a mission is not explicitly modeled in either the QA or LP models. This exclusion is not considered serious because of the times (2 to 7 days) are constant and dependent upon the specific ship. Thus, allowance for this Travel time can be made in the duration of the Inport missions.

The transitions that are assigned low (encouraging) costs are listed. All other transitions are discouraged by high costs.

Inport/All missions
All missions/Inport
Ocean/Alpat

2. Q - Matrix

The Coast Guard Q-Matrix is the same as that of G-G with the specialization that the transition cost proportionality constants equal one for all ships. This matrix serves to associate each time-unit with only the immediately adjacent time-units on the same ship (first and last time-units on a ship are special cases). This structure is the same as that of the Multiple Traveling Salesman problem in the open literature.

3. D - Matrix

The D-Matrix contains the fixed costs of pairing time-unit i to mission-unit j. Five different fixed cost components are used to model five guidelines (The reader can refer to Appendix A where the initial D-Matrix for the sample problem contains labeled examples of each of these components). The first three are the same as in the G-G model.

The first component models the requirements that can be fulfilled by a specific ship. The geographical locations of homeport and mission area primarily determine possible assignments for the GENERAL requirements. This component also models the SHIP-SPECIFIC requirements by allowing only the intended ship to be assigned its SHIP-SPECIFIC missions. If a ship cannot satisfy a requirement, the intersections of all associated time-units and mission-units are assigned very high costs.

The second component provides the mission-to-mission transition cost between the mission assigned to a ship immediately prior to the start of the current schedule period and the first mission it will be assigned in this period.

The third component coerces the enforcement of early start and late finish dates that are associated with each requirement. Each time-unit that follows the late finish time of a requirement is assessed a penalty cost for lateness. This cost increases linearly as the lateness increases. Similarly, a penalty cost for preceeding the specified early start time is assessed. These assessments are made for all mission-units associated with the

requirement. To model the predetermined nature of the first three to six months of the schedule, and the freedom of the last six months, the rate of linear increase of the penalty can be controlled over the time horizon for each requirement. A requirement in the beginning of the schedule can be assigned a higher penalty rate than a requirement at the end. Thus, a deviation of the same amount can be penalized differently, depending upon the proximity to the beginning of the schedule and the flexibility allowed for the requirement.

The <u>fourth</u> component of the D-Matrix does not correspond to one in the G-G model. Some requirements stipulate that the desired duration of the requirement cannot be split and must be completed at one time. This type is usually a SHIP-SPECIFIC requirement. As the QA model allows splitting of requirements into multiple missions, this fourth component attempts to prevent splitting by imposing a structure on the pertinent (time-unit, mission-unit) pairs. Figure 5 shows with and without states for a small example. An additional interpretation is placed on the mission-units; they are to be assigned in a left to right order (the left-most first, and the right-most last). As only one mission-unit can be assigned to each time-unit, the first time-unit after the early start time of the requirement is encouraged to have the leftmost (first) mission-unit assigned to it. The time-unit preceeding the late finish time is also encouraged to have the rightmost (last) mission-unit assigned to it. Similar encouragements are used as appropriate for the remaining mission-units. The encouragement is done by appropriate displacement of the time penalties of the third component.

The effect of this imposed structure is to decrease the number of pairings that incur no time penalty cost. The

(time-unit, mission-unit) pairings are thus encouraged to follow one of the "paths" shown by the arrows. This structure does not guarantee that the requirement will not be split, but does penalize a split.

<u>Single Mission Structure</u>

		<u>Without</u> Mission-Units 1	2	3		<u>With</u> Mission-Units 1	2	3
	1	1	1	1		1	2	3
	2				Early Start		1	2
Time-	3							1
Units	4					1		
	5				Late Finish	2	1	
	6	1	1	1		3	2	1

(Note: The numbers are units of Time Penalty.)

Figure 5

The <u>fifth</u> component of the D-Matrix explicitly models the PATROL requirements (given <u>number</u> of ships on scene at same time). A similar left to right time orientation is placed on the mission-units. By the nature of this type of requirement, a strong relationship is thereby created between particular time-units. That is, the first mission-unit should only be assigned to the one time-unit of each ship that corresponds to the same actual calendar time. Figure 6 depicts the following situation: for a schedule horizon of three weeks (with one week resolution), two ships must satisfy a PATROL requirement of 1 ship always on scene. Thus, the second mission-unit of the requirement is strongly associated with only the one time-unit of each ship that corresponds to the second week of the period. All other time-units are discouraged from possible assignment by high costs (denoted by M).

Patrol Structure

Time-Units	Ship	Time	Mission-Unit 1	2	3
	Ship A	1		M	M
		2	M		M
		3	M	M	
	Ship B	1		M	M
		2	M		M
		3	M	M	

Figure 6

4. Guidelines Not Incorporated in QA

Those parts of the guidelines specifying the morale-related and "limiting" constraints are not modeled in the QA model. No mechanism has been found to condition the assignment of the k^{th} stage on, for example, the amount of AHP time previously assigned each ship. These constraints are incorporated in the subsequent Linear Programming model.

C. LINEAR PROGRAMMING MODEL

1. Objective Function and Constraints

The Linear Programming model for the Coast Guard problem contains the same basic structure as the G-G model. Additional penalties are incorporated in the objective

function tc model the "limiting" and morale-related guidelines. The problem statement of the QA model is broadened. The single target duration of a requirement is extended to have lower/upper limits and the time horizon can vary on each ship.

The primary variables of the LP represent the mission durations. (Note: For the remainder of this thesis, a mission refers to a G-G campaign.) All other variables are non-negative penalty variables that measure the deviations from desired conditions. The six types of penalty variables in the objective function are listed below.

1. For each ship, the difference between the sum of all missions on the ship and the time horizon minus the starting time of the ship.

2. For each ship, the difference between the sum of all Away Homeport missions and the specified total desired amount for that ship.

3. For each ship and each mission-type with a maximum limit, the positive difference between the sum of all missions on the ship for this mission-type and the specified limit.

4. For each mission, the negative difference between the starting time of the mission and the associated requirement's early start time, and the positive difference between the actual start time and the late start time specified.

5. For each mission, the positive difference between the start time plus the duration of the mission and the associated requirement's late finish time.

6. For each sequence of Away Homeport missions, the positive differnce between the sum of durations in the cruise and the limit on the duration of a single

cruise.

The constraints of the model, with no provision for violation at any penalty cost, pertain to the upper/lower limits on the total requirements and the durations of each mission.

1. For each requirement, the sum of durations of the missions associated with the requirement is within the lower and upper ranges.
2. For each mission, its duration is between specified lower and upper bounds.

The PATROL requirements are not handled explicitly. This type of requirement is handled implicitly by the priority assigned to the start/finish times of the requirement. The priority weighting of the linear penalty rate is the same as that explained in the Quadratic Assignment model. Guidelines such as the minimum of eight weeks inport between successive Alpat missions and the minimum cf four weeks prior to an Alpat for preparation are explicitly handled by manipulation of the mission duration bounds.

D. GUIDELINES NOT MODELED

Some facets of the Coast Guard problem have not been analytically modeled. Travel time is not explicitly handled because of the sequencing difficulty already discussed previously in this chapter. The target total Away Homeport time of each ship could be adjusted to partially account for the effect of travel time. The single cruise limit could also be adjusted.

Multi-ship conflicts are not modeled. Overlap of first/last weeks of Reftra, overloading of districts with ships simultaneously in maintenance, staggering the Hawaii-based ships, and alternating winter Alpats are examples. These must be checked and possibly corrected by the scheduler.

The desire to have at least one ship on Inport (search and rescue standby) at all times has not been modeled. Presently, the ship workload is such that this is practically guaranteed. The effects of the new 200 mile fishing limit may prevent this Inport coverage from occurring naturally. Inport can then be handled as a PATROL requirement mission. District training and non-scheduled operations are not modeled since they are not manually scheduled at present.

Single day timing of homeport, return, departure, and on-scene dates must be left to the manual scheduler. Overloading of the fuel pier in Kodiak, Alaska, and Sunday arrivals at Reftra are examples of considerations that the schedulers handle when producing the single day resolution of the published schedule.

The items not analytically modeled can be included in an edit program that would examine the resultant schedule for these relationships and inform the schedulers of hand corrections to be made. Such a utility program could also prove very useful for computing "pseudo-costs" of alternate schedules manually produced.

VI. IMPLEMENTATION

A. OVERAIL CCNSIDERATIONS

The G-G model and computer program described in [4] provide an outstanding example of successful implementation of a complex optimization model for production use. The model concisely captures the pertinent features of the production scheduling problem in an optimization context. The computer system efficiently provides good solutions to the model for the intended client. Unfortunately, the sheer size of the Coast Guard problem makes the G-G system hopelessly expensive to use. Accordingly, a new system has been designed for the large scale Coast Guard problem which preserves all the exciting model features of G-G, while incorporating additional model features and yielding vastly improved computational efficiency.

The primary considerations for the implementation of the analytical model are: to overcome the formidable computer memory and execution time difficulties arising from the large scale of the problem, and to test and evaluate the proposed analytic model. The Geoffrion and Graves code (FORTRAN IV) is used as a starting point. It is designed for commercial use for smaller scale industrial production scheduling problems as discussed in Chapter IV.

An IBM 360/67 at the W. R. Church Computer Center of the U. S. Naval Postgraduate School has been used for development work in this study.

Implementation Flowchart

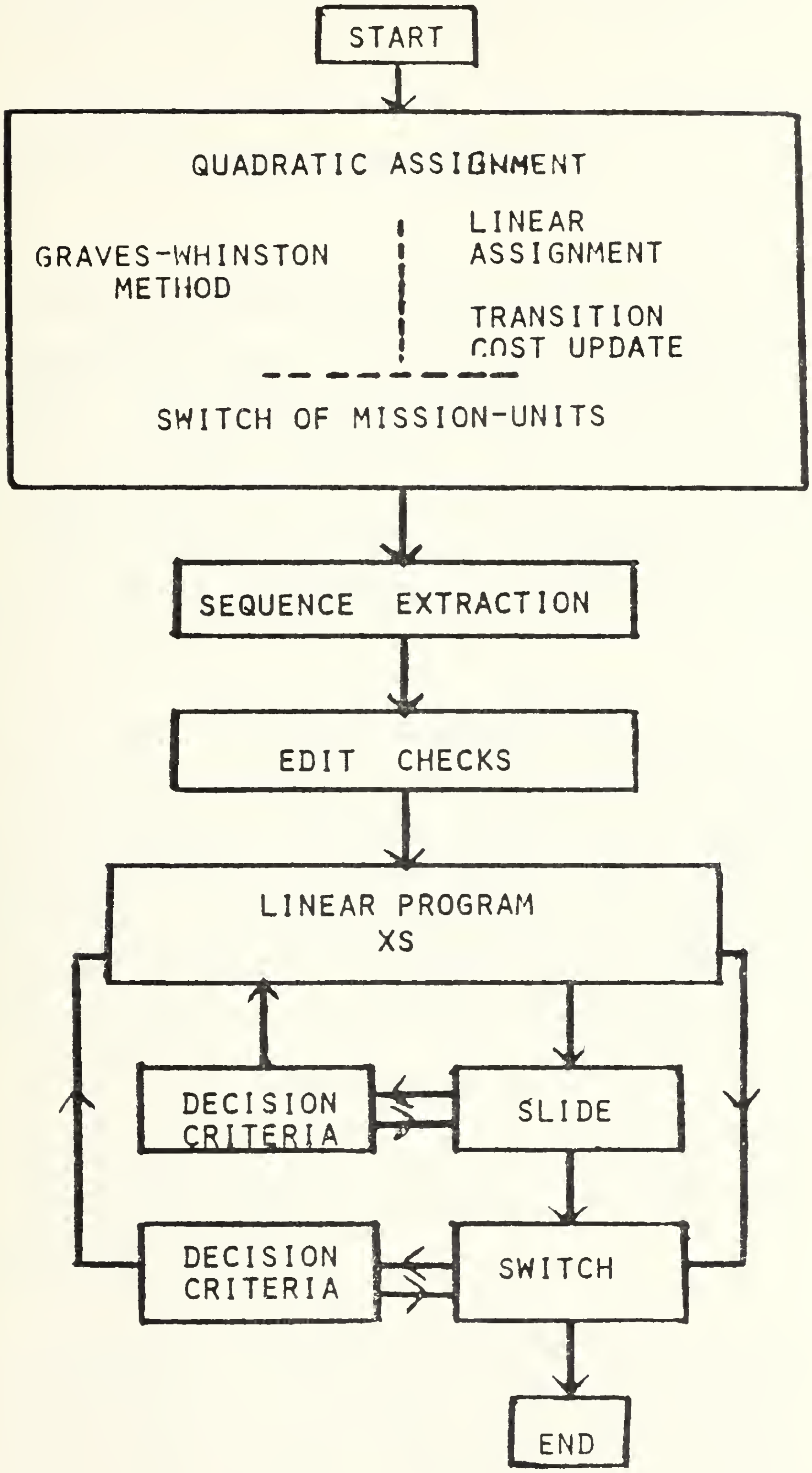


Figure 7

The overall structure of the implementation of the Coast Guard analytical model is depicted in Figure 7.

B. QUADRATIC ASSIGNMENT IMPLEMENTATION

The Quadratic Assignment code used for an initial start is an implementation of the algorithm developed by Graves and Whinston. The code uses explicit in-core storage for each of the three matrices: D, C, and Q. Thus, data storage for a 1,000 unit problem would be approximately 3 million words which is clearly an uncomfortable size. Thus, the first obstacle to overcome to enable the solving of a 1,000 unit quadratic assignment problem is core size.

The G-W algorithm is implemented for the most general case of the QA problem. With the general Q-matrix, many complex relationships between "plants" can be specified (the preceeding/following relationship of time-units is a specialization). The work factor (approximation for numbers of computer operations or expense, as a function of problem size) for the general algorithm is estimated to be on the order of n^4. By specialization of the algorithm to the defined structures of the C and Q matrices of the Coast Guard analytic model, reduction of core and improved performance will result.

The (multiple traveling salesman) specialization used for the Q matrix is to collapse the n*n matrix into two vectors of size n. One vector acts as an index set to give for each row the column number of the single non-zero element (or zero if none is present). The second vector performs the inverse mapping from each column to the row

with the non-zero element, if any. Figure 8 depicts these two vectors for the Q-matrix illustrated in Figure 2 (Chapter IV).

<u>Vector for Rows of Q-Matrix for First Ship</u>

Row i:	1	2	3	...	m_1-2	m_1-1	m_1
Column # of Non-Zero:	2	3	4	...	m_1-1	m_1	0

<u>Vector for Columns of Q-Matrix for First Ship</u>

Column i:	1	2	3	...	m_1-2	m_1-1	m_1
Row # of Non-Zero:	0	1	2	...	m_1-3	m_1-2	m_1-1

Figure 8

For instance, for row 3, the non-zero element appears in column 4; for column 3, the non-zero element appears in row 2. Row m_1 has no non-zero element as it is the last time-unit on the ship (no time-unit follows it). Column 1 has no non-zero element (the first time-unit on the ship has no predecessor).

This change reduces storage requirements from n^2 to 2n, and the work factor from n^4 to n^3.

The special block structure of the C matrix enables the elimination of this matrix resulting in a storage reduction of n^2. By using several levels of indexing from the mission-to-mission transition costs, the appropriate mission-unit to mission-unit transition cost is obtainable

from data arrays already used elsewhere in the model.

The final storage reduction is obtained from the observation that at most two updates are made to each element of the D matrix. After a (time-unit,mission-unit) assignment is made, the fixed cost D matrix is updated. The incurred transition cost into or out of the assigned unit is added to each d_{ij} in the rows preceeding and following the row just assigned. Thus, the maximum value a d_{ij} element can attain is the maximum initial fixed cost plus twice the maximum transition cost. By scaling the initial fixed costs and the transition costs such that the largest possible d_{ij} is less than 2^{15} (32768 is absolute magnitude). Integer*2 (16 bit, half-word) storage is used rather than integer*4 (32 bit, full-word). Integer arithmetic operations are about 10% slower than floating point arithmetic on the machine used in this study, but the data storage reduction has to be made.

By these changes, the total data storage requirement for the three matrices is reduced from $3n^2$ words to approximately $n^2/2$ words. Thus, a problem with 1,000 time units would require 500K words rather than 3,000K words for data storage.

From trial runs, three additional algorithmic specializations have been noticed for the Coast Guard

problem. Mission-units for Inport comprise one fourth to one third of the total number of mission-units. All mission-units of Inport have identical initial fixed costs. All dynamic transition cost updates maintain this identity. Thus, during the expected value calculation matching each unassigned time-unit with all unassigned mission-units, only one of the unassigned Inport mission-units need be tried as a candidate rather than all n/3.

The second specialization arises from the observation that almost all of the Inport mission-units are always the last mission-units assigned to time-units and that the expected value of the future cost of these new assignments is zero. Once the point is reached where only Inport mission-units remain unassigned, the identical nature of all of these units means that the expected future cost of their assignment becomes permutation independent. These units can be assigned to any unassigned time-units and in any order.

The third specialization depends on the likelihood that infeasible (strongly undesired or impossible) assignments occur. Prior to calculating the expected value of assignment of (time-unit i, mission-unit j), the fixed cost for this assignment, d_{ij}, can be tested. If this fixed cost has exceeded a predetermined infeasibility value, the trial assignment can be preemptively skipped. A difficulty arises if an infeasible assignment should really take place via the lowest mean criterion. With this efficiency-motivated modification, abnormal termination with an incomplete assignment map would then occur since the algorithm would bypass this infeasible assignment.

These three specializations have not been made for reasons apparent later in this Chapter (a new algorithm for the QA model is developed). It is speculated that their

affect will reduce the work factor of the specialized G-W method from n^3 to $n^3/4$ if Inport missions comprise one third of the schedule.

Following the completion of a (time-unit,mission-unit) map by minimum expected values, an additional algorithm (called "Switch") is tried to obtain quickly and easily further improvement in the objective function. All pairwise interchanges of the mission units are cyclically tested by exhaustive enumeration. For this Coast Guard problem, striking improvements are achieved. This effect and other basic performance data of the G-W method is listed in Figure 9. (The 5 day time resolution has not been run because the execution time would be excessive.)

C. A NEW QUADRATIC ASSIGNMENT METHOD

The Graves-Whinston method, with the modifications and specializations described above, is estimated to use four to eight hours of computer time (IBM 360/67) to solve a 1,000 unit Quadratic Assignment problem. Since this is excessive (especially considering that the purpose of the overall analytical model is ultimately to aid manual schedules in an interactive fashion), a second method of implementation has been explored. Other reasons for introducing a second method are:

1. The "limiting" and morale-related constraints are not modeled in the QA;
2. The QA sequences will be subject to increasing modification as the LP model incorporates these new types of constraints;
3. Significant cost reductions are made by the Switch

transformation; and

4. The fixed cost matrix for the Coast Guard problem is highly structured.

For these reasons, the new method is designed to quickly obtain a feasible assignment of (time-unit i, mission-unit j) pairs using only the fixed cost D-matrix. A Linear Assignment problem is solved to obtain a set of pairings with the minimal total fixed cost. Then, the fixed cost matrix is updated with transition costs in the same manner as the G-W method. The same Switch algorithm is then used to minimize total fixed and transition costs. The final solution, obtained at the end of this algorithm, is a solution to the original Quadratic Assignment problem.

The Linear Assignment problem is solved by a special subroutine called CGNET, which is an adoption for the Coast Guard problem of the well-known primal simplex network package GNET [1]. The package is modified to use the completely dense assignment (fixed) cost matrix and to exploit where possible the special structure of these extremely large problems. Several external parametric controls permit tuning of the package for efficient performance.

The network problem for the 5 day resolution has 860 nodes and 739,600 arcs. CGNET constructs an optimal Linear Assignment solution in 11.3 minutes (IBM 360/67). A literature search indicates that this is the largest assignment problem for which an optimal solution has been constructed. The previous record was a 450,000 arc model done in a feasibility study for the Navy Computer Assisted Distribution and Assignment (CADA) model.

The memory requirement of $n^2/2$ words for data storage is the same as the G-W method's. Figure 9 shows the comparison between the G-W and CGNET methods for computation time, effectiveness and computational cost of the Switch algorithm, and total pseudo-cost of the final (time-unit, mission-unit) pairing. Note that CGNET produces excellent quality solutions with significantly less computational expense.

<u>Performance of Quadratic Assignment Methods</u>

Resolution	20 DAY		15 DAY		10 DAY		5 DAY	
Size (n)	206		288		420		860	
Method	G-W	CGNET	G-W	CGNET	G-W	CGNET	G-W	CGNET
Assignment Time (min)	8.23	0.38	20.78	0.90	69.00	2.37	--	11.32
Switch Time (min)	0.27	0.42	0.52	0.92	1.55	1.91	--	7.66
Total Time (min)	8.50	0.80	21.30	1.82	70.55	4.28	est 480.	18.98
Assignment Cost	81110	148730	96120	172740	110060	182070	--	168230
Final Cost	37470	52570	38590	36480	45960	40940	--	11100
% Switch Improved	53.8	64.7	59.9	78.9	58.2	77.5	--	93.4

Figure 9

D. POST-PROCESSING OF THE QA SOLUTION

The Graves-Whinston and CGNET methods for the QA problem do not guarantee that the optimal solution will be found. In addition, the solution may contain infeasible assignments, multiple campaigns when only one is desired, and illegal transitions. To determine if these undesired events have occurred, quick and effective edit checks are conducted.

In the hybrid approach of the analytic model , the Quadratic Assignment solution is examined and used for only the pure sequence of missions conducted by each ship. The consecutive time-units of a ship are checked for each sequence of mission-units associated with the same requirement. Checks are made for assignment of missions to ships for requirements that they cannot fulfill. Any such mission that is found is relocated to be the first mission on the first ship able to conduct the mission. The structure placed in the D-matrix to encourage a single mission for a requirement when this is desired does not guarantee this result. Thus, an edit check that retains the first mission encountered and discards the rest is performed. Another quality not guaranteed by the QA solution is the absence of all infeasible or undesired transitions, such as Alpat/Reftra. An optional edit check can eliminate this type of transition by the insertion of an Inport mission wherever needed. However, as later procedures may cause the reintroduction of such transitions, this check is best delayed until after the final LP solution is obtained.

E. IMPLEMENTATION OF LINEAR PROGRAMS

In this hybrid model, once the mission sequence is obtained, a linear program is solved to determine the mission durations that minimize the objective function. Then, neighboring mission sequences are generated and tested to see if improvement occurs. A neighboring sequence is defined as any sequence that is generated by applying one of the following operations to the present sequence:

1. Move one mission to another position (called Slide).
2. Interchange the position of two missions (called Switch).

This trial of neighboring sequences is absolutely critical for this Coast Guard problem because of the differences in the content of the QA and LP models for this problem. The QA solution's sequence will be far from optimal because the "limiting" constraints are not considered. Many neighboring sequences will be favorable candidates to reduce total cost and for each such neighboring sequence, an LP solution is necessary. Thus, the computational requirements, core and time, must be minimized for each LP call.

The initial LP code (used in the G-G implementation) solved a problem of 330 rows and 330 variables in the range of 17 to 21 seconds on the IBM 360/67. Analysis of the constraint equations shows that only 1% of the total coefficient matrix is non-zero and the only non-zero value is +1. Thus, an LP code using a data structure for the sparce constraint coefficients is desired. Also, a code allowing a reduction in the number of equations by ranging and bounding methods reduces the dimensionality of the

problem and thus reduces computational time.

Since the QA and LP portions of the hybrid model are distinct, the program and data areas of each model can be overlayed in main memory, resulting in significant savings of core. With a projected LP problem size of 300 rows and 500 variables, the original LP code would have storage requirements larger than either specialized QA code for time resoluticns cf 20 days or less.

A new LP system called XS has been installed for use with the Ccast Guard problem. XS [2] is a prototype optimization system designed to serve as a research testbed for evaluaticn of advanced design features for large scale linear, nonlinear and integer problems. The system is used as a subroutine, with the LP problem set up provided by the calling discipline. Salient features of the version of XS employed include implicit ranging (upper and lower values for each ccnstraint), logical bounding (upper and lower values for each variable), and effective external controls for tuning the system to perform well on the class of problem at hand.

The LP cbjective function, described in Chapter V, is composed cf non-negative penalty variables whose values measure the deviation of the schedule from the desired goals. The deviation is obtained by expressing the goal as a constraint, with the insertion of the appropriate slack or surplus penalty variable in the equation or inequality.

For example, the deviation of a ship from the desired total AHP time is determined by adding the following constraint tc the LP:

Objective Function: (cost * slack) + (cost * surplus)

Constraint:

$$\text{Total AHP Limit} = \sum \text{All AHP Missions} + \text{Slack} - \text{Surplus}$$

Thus, the implementation of the LP model has the two types of non-violable constraints and six penalty-related constraints (that determine the value of the penalty variables). Several of these types of constraints are independent of the overall number and structure of the sequence of missions. These are the total horizon time, AHP time and Alpat time on each ship, and the early/late start and late finish time penalty constraints. The number of time penalty equations is fixed by the number of missions for requirements possessing these penalties. Since no missions are deleted or added by the LP, no change in the number of missions occurs.

The number of equations for the cruise duration penalty and the quantity bounds on the requirements can be reduced for each individual LP based on its particular mission sequence. The number of Away Homeport cruises is dependent on the individual sequence. For each sequence of two or more AHP missions, an equation is introduced. Rather than introduce an equation for a solitary AHP mission, the mission duration's upper bound is compared to the AHP cruise limit and lowered to this value if necessary. This method does not allow a solitary mission to exceed the cruise limit, even for a penalty assessment. (This is a change to

the strict definition of the LP model given in Chapter V.)

The last constraint type is the quantity restriction on each requirement. These are hard constraints with no provision for penalties for non-compliance. An equation is introduced only if a requirement is split into two or more missions. If only one mission produces the requirement, all previous bounds on the mission duration are superceded by the minimum and maximum quantities of the requirement. Note that if the single mission forms a solitary AHP cruise and the maximum quantity exceeds the AHP sequence limit, no penalty will be charged if the cruise limit is exceeded as no equation for cruise penalty is generated.

The typical LP for a 6 ship, 2 year schedule has 171 rows and 435 variables. The number of rows for each type of constraint is: 6 each for total horizon, total AHP time and total ALPAT time; 86 for early/late start times; 51 for late finish times; 12 for cruises of 2 or more missions; and 4 requirement duration totals. There are 145 mission variables and 290 penalty variables.

The nature and balance of the objective function is critical to obtaining the desired type of schedule. The balance between the six types of penalties will determine the relative importance and preferences of the tradeoffs that will be made during the minimization process.

The cost structure for each type of penalty is also an important factor. Figure 10 depicts the present structures.

Present Penalty Cost Structures

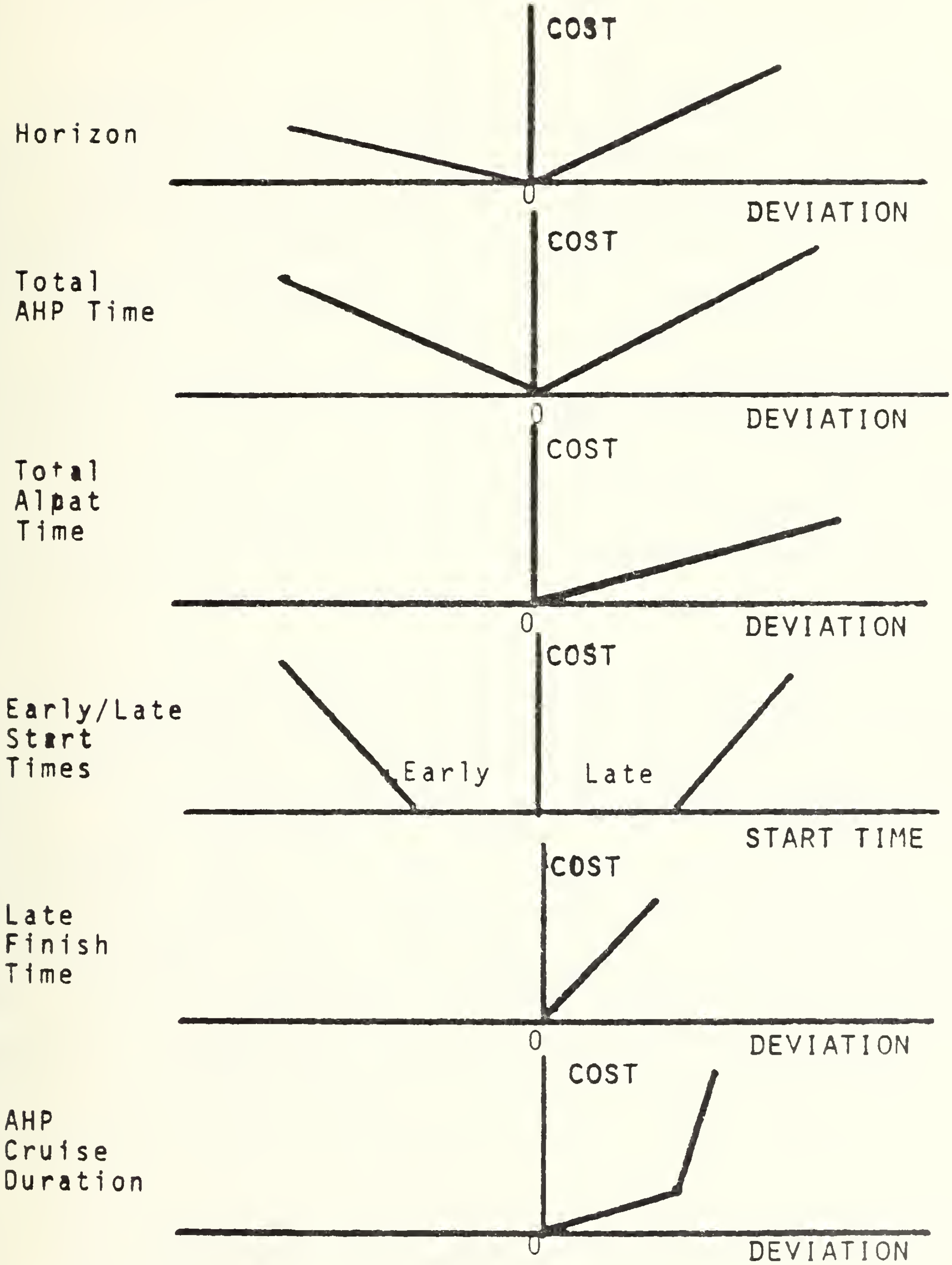


Figure 10

F. IMPLEMENTATION ALTERNATIVE FOR LP

Examination of the solutions to all test LP's shows that mission durations were always exact integers. This leads to the investigation of a Network formulation of the LP. A complete transformation to a network has not been found for the CG model because of a few complicating constraint types. The horizon limit, time penalty and requirement quantity constraint types (which collectively comprise the entire original G-G model and about 85% of the equations of this Coast Guard model) can be reformulated as a pure network model. This reformulation has not been implemented, but a significant reduction in time for each LP call, especially for the G-G model, would occur.

The variables (Arcs) for the network formulation are the length and the starting time for each mission. Figure 11 pictorally shows the fundamental structure of the network. There is one equation (node) for each mission of a requirement that has any of these qualities: a late completion penalty, a following mission with an early start or finish date. Each requirement with multiple missions will have an equation. Single mission requirements are handled by appropriate bounds on the mission's length. The total time limit equation for each ship is added between the last mission node for each ship and the root node.

The balancing of AHP time is handled in the network by its complement, the time spent on Inport missions. Three types of missions emerge from the root node. The first type is all AHP time requirements. The second type is all SHIP-SPECIFIC homeport requirements. The third type is all GENERAL ship in-homeport requirements, usually only the

Inport (SAR standby) requirement that is the system's slack. To obtain the total non-AHP time on a ship, the type two and three missions on a ship need to be added in some manner. If the type two missions do not have a fixed total, this can not be done by the network. If they do have a fixed limit, appropriate limits and penalties can be placed on the sum of type three missions going to each ship. Actually, calculations to check for this situation may not be worthwhile in actual implementation. Replacement by one non-network equation per ship is simpler. Non-network equations are also necessary for each AHP cruise sequence of 2 or more missions and one per ship for each mission-type with maximum limits (i.e. Alpat).

An important property of the network is the resulting pictorial display of the model. Based on this picture, a method to perform dynamic changes to the formulation may be developed to eliminate the complete problem generation for each new LP mission sequence. Alternatively, a mission that is a candidate for relocation may be placed in several places at once. Sensitivity analysis and reoptimization can determine the one best placement faster than a succession of complete LP calls. This and an ability to reoptimize based on the current solution may drastically reduce the time and increase the information gained from each LP call.

NETWORK FORMULATION

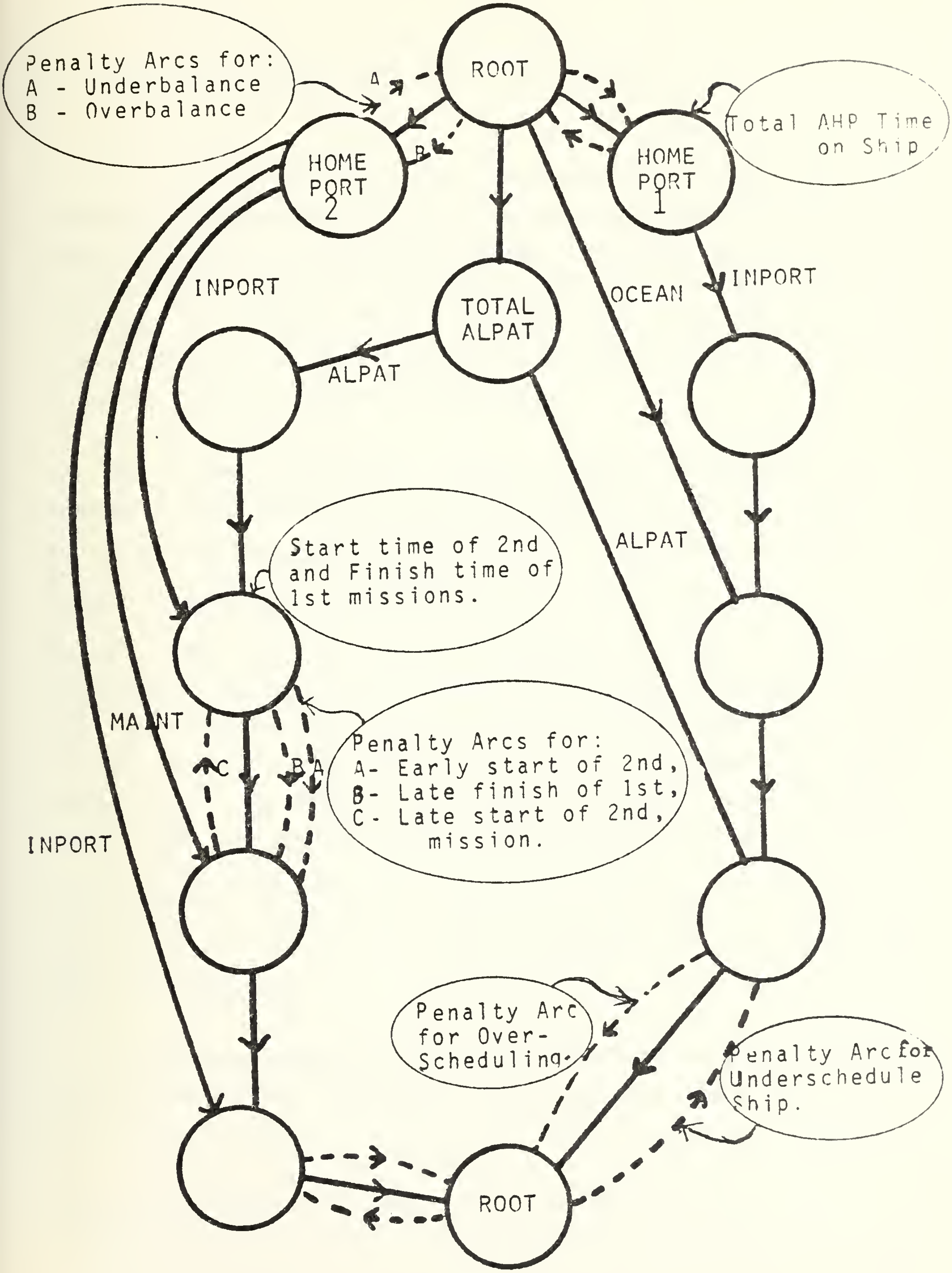


Figure 11

G. CALLING STRATEGY FOR THE LINEAR PROGRAMS

The first sequence of missions obtained from the Quadratic Assignment solution and the post-QA/pre-LP edit processing will be far from optimal because the "limiting" constraints are first introduced in the LP model. Thus the ability to improve the solution by evaluating many "neighboring" mission sequences is vital to the successful solution of the scheduling problem.

The first algorithm (called Slide) performed on the mission sequence removes each mission in turn and inserts it into all other possible locations. The number of different mission sequences to be evaluated in some manner is n^2. An actual call for a full LP solution, that is, complete explicit enumeration, is clearly impossible. A calling strategy implicitly evaluates each possible sequence and an LP is solved only for the most promising candidate sequences. Additionally, after a neighboring sequence is found with an improved total sequence and LP penalty costs, it becomes the incumbent solution and another n^2 cases are evaluated until no further improvements can be found with this transform.

The second algorithm (called Switch) evaluates pairwise interchanges of missions. The number of possible switches for each incumbent sequence is $(n^2-n)/2$. Again, after an improving switch is found, the transform is restarted.

For each type of transform, a calling strategy to decide

when to invest in an LP solution is critical. Those transformed sequences that introduce assignments on ships incapable of fulfilling the requirement can easily be detected. But for feasible assignments, the expected change in each of the six types of cost penalties must be calculated or estimated. The change in the sequence transition cost is deterministic and easily calculated. The changes in the penalties, however, have an unpredictable nature because they are dependent on the mission lengths which are set by the LP to obtain the minimal objective function.

To solve this problem for Slide, a detailed calling strategy is implemented for five of the penalty types. (The horizon penalty is not included.) The transition cost is deterministically found and estimates are made for the change in start and finish time, total AHP, total Alpat, and AHP cruise penalties. Amazing accuracy has been achieved. In the early stages of the algorithm, a high percentage of the LP calls that are made lead to improvements extremely close to the prediction.

Other methods to maximize the contribution of each LP call are theoretically similar to pivot pricing and selection rules. For Slide, a mission is moved to each feasible new position and the expected cost change is calculated. The proposed new position with the most beneficial expected change is the only one considered further. The predicted change is then compared to a decision rule cutoff value. Based on this test, an LP call is either made or the Slide algorithm discards that mission as a candidate and proceeds for the next mission.

In another strategy, the decision cutoff value is changed as the algorithm progresses. Presently, the sliding decision rule has four values, each higher in value (-8000,

-4000, -1000 and -600). Those sequence changes that cause very large and beneficial cost improvements are actively sought in the opening stages. After one decision rule value leads to a complete circuit of Slide with no improvements being found, the rule assumes the next higher value and the algorithm is restarted. The computational time spent doing these calculations and tests for a complete circuit of the algorithm is less than half the time of a single LP call. Figure 12 shows the dramatic improvements made in the objective function, the percentage of LP calls being made that lead to true improvements, and the overall success of these calling strategies for Slide.

For the second algorithm, Switch, a prediction of the cost change is made for only the start and finish time penalties and the total AHP time penalties. The transition cost change is deterministic and, thus, is calculated. Only one value of the decision rule cutoff value is used. LP solutions are obtained for all favorable candidate sequences rather than for only the most favorable. The performance of this algorithm is also indicated in Figure 12.

LP Calling Strategy Performance

Slide and Switch Algorithms

Slide: 4 levels of decision criteria: -8000, -4000, -1000, -600

Switch: 1 level of decision criteria: +10

If the estimated solution improvement is less than the current decisicn criteria, the LP is formulated and solved.

Definition:

Hit Ratio: = # LP's with improvement made / Total # LP's called

Time Resolution		20 DAY	20 DAY	20 DAY	20 DAY
Initial Cost		348500	319000	348000	810000
SLIDE:	Cost End of -8000	109000	112000	121000	174000
	Hit Ratio	7/7	6/6	6/6	10/10
	Cost End of -4000	95900	90000	101000	149000
	Hit Ratio	2/2	4/8	4/8	6/20
	Cost End of -1000	91600	70000	81000	122000
	Hit Ratio	4/20	9/20	10/23	11/49
	Cost End of -600	88630	63000	71000	121650
	Hit Ratio	2/70	6/13	9/51	1/31
SWITCH:	Cost End of +10	82650	62700	68416	98740
	Hit Ratio	7/30	0/3	3/22	15/59

Initial Cost	348500	319000	348000	810000
Final Cost	82650	62700	68416	98740
Composite Hit Ratio	22/129	25/50	32/110	45/169
Average Time/ LP (secs)	5.32	6.50	4.79	4.47

Figure 12

VII. RESULTS

A. USE OF THE MODEL

Before discussion of the qualitative properties of the schedules produced by the implementation of the Coast Guard analytical scheduling model, the general types of input data and user external controls must be understood. There are seven types of input data used by the model:

1. Number of ships, number of mission types, the time span of the schedule, and the time resolution of the QA model.

2. Description of the mission types: away/in homeport, single (no split of requirement), and/or Patrol.

3. Description of the ships: capability for each mission type, last mission on previous schedule, initial fixed portion of present schedule.

4. Description of requirements: mission type, target demand for QA formulation and lower/upper limits for LP formulation, early and late start times and late finish time, priority of times (for control of time penalty rates).

5. Pseudo-Costs: mission-to-mission transition costs and cost rates for the penalty variables (horizon, start/finish times, total AHP time, total Alpat time and cruise duration).

6. Desired goals for total AHP time, total Alpat time, and cruise duration.

7. Implementation controls: LP calling strategy decision cut-off values, level of output, length of maximum run.

By the specification of the input data, the scheduler has effective control of the resulting schedule. The cost rates for the penalty variables determine the balances and tradeoffs that will be made by the optimization process to arrive at the final proposed schedule. By adjusting a rate, the influence and importance of that guideline's goal is changed with respect to all others. An example of cost balancing between transition costs and the cruise penalty rate is given in Figure 13.

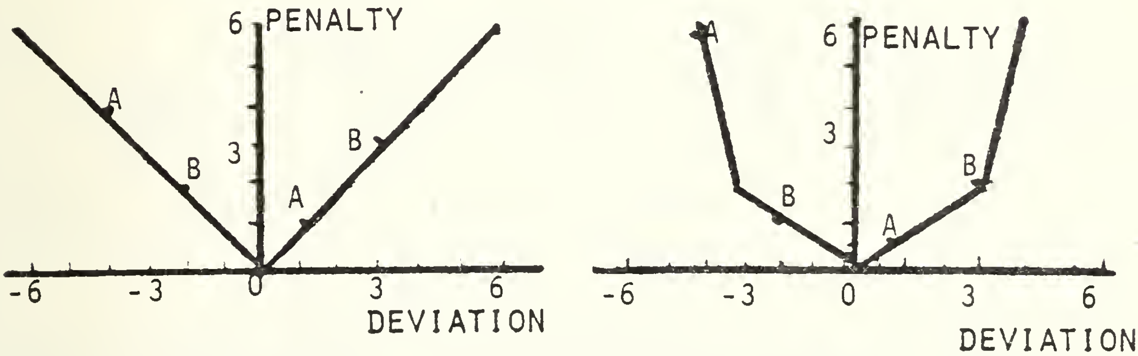
Another important control is provided by the cost penalty structures shown in Figure 10 (Chapter VI). The influence this structure has on the tradeoffs and preferences within each guideline goal is illustrated in Figure 14. A structure with larger penalties for large deviations forces the AHP balance closer to the desired goal.

A powerful property of the hybrid QA/LP approach is the separation of the combinatorial sequencing of missions for each ship and the determination of the duration of each mission. This approach and the implementation techniques provide the capability of bypassing the QA, using a previously generated sequence, setting the penalty rates and obtaining a potential schedule. (Note: Since the QA model does not include the "limiting" and morale-related constraints, changes in the penalty rates or desired goals for these constraints affect only the LP portion of the model.) Dynamic interactive use of the model is an inherent property of the modeling approach.

Influence of More Breakpoints in Penalty Cost Structure for Total Away Homeport (AHP) Time

Two Alternatives for Distributing Total AHP Time

Set A:	Ship 1:	four units under desired balancing amount
	ship 2:	one over
Set B:	Ship 1:	two under
	Ship 2:	three over



Total Penalty

Set A: $4 + 1 = 5$
B: $2 + 3 = 5$

No preference between the sets exist from Cost structure.

A: $6 + 2/3 = 6\ 2/3$
B: $4/3 + 2 = 3\ 1/3$

Cost structure prefers B over A. B is more central about desired total amount.

Figure 13

Balancing of Costs
Between Two Components

Situation: Schedule 10 weeks Alpat
2 weeks Ocean
Ignore all penalties except Cruise limit and Transition costs.
Cruise limit is 10 weeks.
Alpat and Ocean are AHP missions.

Alternatives:

A Inport/Ocean/Alpat/Inport
B Inpcrt/Ocean/Inport/Alpat/Inport

Cost Structure I

Transition Costs

	Inport	Alpat	Ocean
Inport	0	10	10
Alpat	10	0	1000
Ocean	1000	10	0

Cruise penalty
2000/unit over 10

Ccsts:	Transition	+	Cruise	=	Total
A:	30	+	4000	=	4030
B:	1030	+	0	=	1030

B is preferred but as discussed in a previous Chapter, Inport/Ocean/Inport is an infeasible sequence. The penalty cost of Ocean/Inport should be more than that necessary to link Ocean/Alpat even if cruise limit is violated.

Cost Structure II

Transition Costs

	Inpcrt	Alpat	Ocean
Inport	0	10	10
Alpat	10	0	1000
Ocean	7000	20	0

Cruise penalty
2000/unit over 10

Costs:	Transition	+	Cruise	=	Total
A:	30	+	4000	=	4030
B:	7030	+	0	=	7030

A is preferred, as desired by the guidelines. The cost of Ocean/Alpat was raised above the increase in cruise penalty caused by adjoining 2 weeks of Ocean and 10 weeks of Alpat.

Figure 14

The model can be used for both strategic and tactical decisions. Strategic "what if" questions can be examined for their effects on scheduling (for example, changes in the number of ships available, the cruise limit for a new class of ship, or new areas of responsibility for the Coast Guard). Dynamic schedule changes, due to requirement changes or unanticipated ship non-availability, can be evaluated and the least "upsetting" solution found.

B. QUALITY OF SCHEDULES PRODUCED

Because of the importance of managerial judgement and personal preferences, no single MOE is available to quantitatively measure the quality of the schedules produced by the analytic model.

The schedules produced completely satisfy the restrictions governing desired and undesired mission transitions. No transitions with high, infeasible costs are present. Almost all requirements are scheduled to meet their specified start and finish times. Those that are not Alpat missions are usually one or two weeks displaced from the desired times. The Alpat missions, having the Patrol requirement, are exceptions. As explained, the Patrol requirement is implicitly modeled by setting high priority (causing a higher penalty rate) on fulfilling the specified start/finish times. About 70% of the Alpat missions are scheduled to satisfy the times, but the remaining Alpat missions vary from being displaced 1 week to a maximum of 8 weeks.

This result for a Patrol requirement mission-type is not realistic. To obtain even this result, it is necessary to

express the overall requirement as many detailed, individual requirements with exact durations (equal lower/upper bounds) and specific start/finish times. This level of input detail is an undesirable feature. This result and the necessary detail of data are implementation issues, and not model shortcomings. model. An enhancement discussed in Chapter VIII will correct this poor performance.

With the exception of the Patrol type of mission, the present model produces good quality solutions. The guidelines that are explicitly modeled and those that can be implicitly modeled by the scheduler through the input data capture all of the major considerations of the overall guidelines. This model also meets the objective of quickly generating several potential schedules. Minor variations in some control parameters, particularly the minimum and maximum durations for Inport missions, can cause alteration of the schedules. For the Coast Guard problem, this ability to easily generate alternatives is a highly beneficial property.

The model, in summary, provides good quality solutions, generates alternative schedules easily, and is highly responsive to managerial controls. This model can be a powerful managerial tool for scheduling the High Endurance Cutters.

The use of this model for Medium Endurance Cutter scheduling (not the direct focus of this study) appears more in doubt: principally because shorter ship endurances cause shorter patrol lengths; more multi-ship relationships exist for Search and Rescue coverage; Reserve training cruises are highly complicated; and larger relative lengths of travel time to patrol time exist.

C. QA / LP RESULTS

The size and computational difficulty of the Quadratic Assignment problem is determined by the time resolution. The time resolution (S) is the unit that determines the number of finite time-units to be scheduled for each ship, and the number of mission-units for each requirement. The results displayed in Figure 9 (Chapter VI) show the computational performances of the Graves-Whinston and CGNET methods for time resolutions of 20, 15, 10 and 5 days.

Comparison of these methods and/or resolutions cannot be based completely on the final QA solution cost. Between resolutions, the number of mission-unit to mission-unit transitions is different, causing differences in transition cost. Between methods, the final cost is an indication of the degree of compliance of the schedule to the desired mission sequences and start/finish times. It is not an indication of the quality of the overall solution that will result at the end of the LP since the "limiting" constraints are not modeled in the QA. Also, the contribution of the QA is only the mission sequence extracted from its (time-unit, mission-unit) pairs, not the additional timing information.

The CGNET method usually produces a larger number of separate missions than the G-W method. Most of the additional missions are Inport which is beneficial later in the "neighboring" sequence searches. Having more Inports causes larger transition cost reductions to be possible.

Qualitative evaluation of the model's final schedule shows that the method of solution of the QA has no impact on the final quality. Each method produces a superior final

schedule for about half of all cases. CGNET, with its good computational performance, is the recommended method.

Contrary to expectations, the 20 day resolution provides "higher" quality overall schedules than the smaller (5 day, 10 day) resolutions; the 5 day resolution produces the worst schedules. The schedules from the finer resolutions have greater difficulty in satisfying the transition restrictions and reducing the violations of the desired start/finish times.

The computation time for each LP solution is primarily determined by the number of missions. With about 145 missions, an LP requires, on the average, 0.5 seconds for problem generation, 4.5 seconds for solution, and 100 pivots. The LP calling strategy implemented is of vital importance - the most significant improvements are located and made first. In this way, the total number of individual LP solutions is minimized. In all experiments, the schedules with the lowest pseudo-cost require fewer total calls to the LP, and have the highest hit ratios. This result means that after the first 20 to 30 significant improvements, extended computer time and more LP calls are not particularly effective.

To summarize the computational performance, the best schedules are obtained by the CGNET method for the QA with 20 day time resolution. Computation time for this method is about 52 seconds. With various bounds on Inport missions, the LP portion of the model requires an average of 7 minutes, 36 seconds. Thus, good potential schedules are obtained by this analytic model in an average 8.5 minutes using 70 K words of memory.

VIII. ENHANCEMENTS

A. APPROACH

With the overall success of the hybrid approach, major changes in the modeling approach are not necessary. With the exception of the PATROL requirement, the items discussed in this Chapter are ideas for possible improvement of computational performance and additional managerial controls that would be useful in a production level implementation.

Improvement (fewer total LP calls) would result if the "limiting" and morale-related constraints can be considered in the QA. The inclusion of these constraints requires a mechanism to condition each assignment (on the total number of AHP time-units previously assigned each ship, for example). The Graves-Whinston method, which uses probability in determining the next assignment, can potentially be extended. So far, the necessary theoretical developments in this area have not been made.

The Linear Programming model needs explicit modeling of PATROL requirement missions. A new type of penalty or non-violable constraint linking the end of one ship's patrol and the beginning of the next ship's should be added. (The next section gives further discussion of this point.) Also, additional equations for AHP balancing for each year as well as the whole schedule horizon may be needed. A difficult decision is where to split the mission sequence into those AHP missions of the first year and those of the second year

prior to the mission durations being known.

The introduction of human interaction at several points may also be worthwhile. Visual inspection of the sequence from the QA solution, as well as periodic inspection of the progress of the LP calling strategy and sequence searches may improve computation times and solution quality. The addition of a post processor to inform the scheduler of the status of those guidelines not analytically modeled would also be appropriate for a production level program.

B. IMPLEMENTATION

Several implementation modifications will increase managerial control and further enhance computation speed. A more complicated selection scheme for deciding which one of multiple missions to retain for requirements that cannot be split would remove at least 10 to 15 LP calls. Also, relocation of infeasibly assigned missions to the best rather than the first location would be an improvement. As shown by Figure 14 (Chapter VII), a change to a more convex penalty structure for total AHP time would lead to better workload balancing between the ships. Travel time can be explicitly modeled in the LP model. The travel times between areas for each mission transition are known. The lower/upper bounds on mission durations and the desired total time for each ship can be dynamically calculated during the generation of each LP at no increase in LP solution time. All of the above listed ideas are easy modifications to the present implementation.

A more difficult implementation modification is the explicit modeling of missions with the PATROL requirement in the LP model. The alternate network formulation discussed

in Chapter VI most easily accomodates this change. In this formulation, each mission's starting time and duration are available as primary variables. The linking of one mission's finish to another mission's start is easily and directly acccmplished. This enhancement will remove the only difficulty with the quality of the final proposed schedules, and also remove the undesired detail presently required in the input specification of PATROL requirement missions.

APPENDIX A

SAMPLE PROBLEM AND RESULTANT SCHEDULE

Problem: Two ships are available for a 7 week schedule horizon to fulfill these requirements:

1. Seven weeks of Alpat with PATROL requirement;
2. Two weeks Maint for Ship Two between weeks 5 and 6;
3. Two weeks of Ocean between weeks 2 and 5; and
4. Three weeks of Inport.

Alpat and Ocean are Away Homeport missions. Only Ship One can fulfill the Ocean requirement in this period. The last mission on the previous schedule for Ship One is Inport; Ship Two, Ocean. The time resolution for the QA model is one week. The following list contains the cost penalties and desired goals:

1.	Infeasible assignment cost	700.
2.	Time penalty cost rate	50.
3.	AHP penalty cost rate	600.
4.	Cruise penalty cost rate	2000.
5.	AHP desired goal per ship	4 weeks
6.	Cruise limit	5 weeks

Transition Costs:

	INPORT	ALPAT	OCEAN	MAINT
INPORT	0	10	10	10
ALPAT	10	0	700	700
OCEAN	700	10	0	700
MAINT	10	700	700	0

Initial D-Matrix of Fixed Costs

0	0	0	10	350	350	350	350	350	350	60	110	700	700
0	0	0	0	0	350	350	350	350	350	0	50	700	700
0	0	0	350	0	0	350	350	350	350	0	0	700	700
0	0	0	350	350	0	0	350	350	350	0	0	700	700
0	0	0	350	350	350	0	0	350	350	50	0	700	700
0	0	0	350	350	350	350	0	0	350	100	50	700	700
0	0	0	350	350	350	350	350	0	0	150	100	700	700
700	700	700	10	350	350	350	350	350	350	700	700	700	700
0	0	0	0	0	350	350	350	350	350	700	700	150	150
0	0	0	350	0	0	350	350	350	350	700	700	100	100
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0	0	0	350	350	350	0	0	350	350	700	700	0	0
0	0	0	350	350	350	350	0	0	350	700	700	0	0
0	0	0	350	350	350	350	350	0	0	700	700	50	50

Illustrated Cost Components:

1. Infeasible Assignment
2. Transition from previous schedule
3. Time Penalty
4. Single Mission Structure
5. PATROL Structure

Assignment Map:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	11	12	7	2	8	9	4	3	5	6	14	13	10

Assign Fixed Cost = 10
Transition Cost = 1460
Total = 1470
Enter Switch 1470 Leave Switch 410

Assignment Map:

1	2	3	4	5	6	7	8	9	10	11	12	13
1	11	12	6	7	8	9	4	5	10	2	14	13

Schedule 1

Ship One Mission	Weeks
Inport	1
Ocean	2,3
Alpat	4,5,6

Ship Two Missions	Weeks
Alpat	1,2,3,4
Inport	5
Maint	6,7
Inport	8

Ship One		Ship Two	
Transition Costs	= 20	Transition Cost	= 40
Penalties: Horizon	= 50	Penalties: Horizon	= 50
AHP	= 600	Time (Maint)	= 50
	670		140

Total = 810

Schedule 2

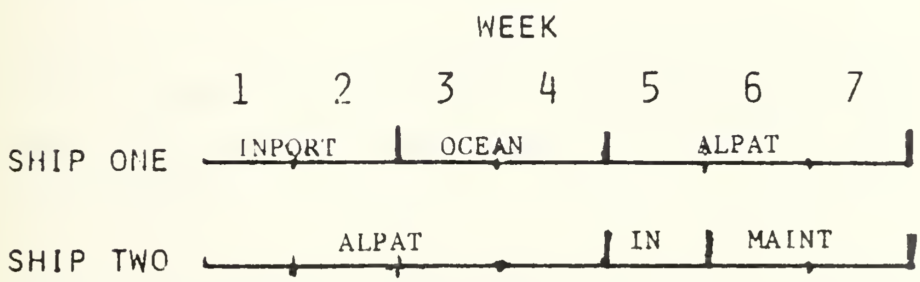
Ship One Mission	Weeks
Inport	1,2
Ocean	3,4
Alpat	5,6,7

Ship Two Missions	Weeks
Alpat	1,2,3,4
Inport	5
Maint.	6,7

Ship One		Ship Two	
Transitcn Cost	= 20	Transition Cost	= 30
Penalty: AHP	= 600	Penalty: Time (Maint)	= 50
	620		80

Total = 700

FINAL SCHEDULE



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